THE CLASSICAL THEORY OF MONEY

Exploring the foundations of monetary economics through Walrasian general equilibrium and the Cambridge equation of exchange.





Bethlehem, 19th Century



London, 19th Century



Lucknow, 19th Century



London or Paris, 19th century



LEON WALRAS (b.1834-d.1910)

FRENCH-SWISS MATHEMATICIAN/ECONOMIST

FOUNDER OF GENERAL EQUILIBRIUM THEORY

"In a theoretical work that assumes a 'regime of perfectly free competition,' Walras constructed a mathematical model in which productive factors, products, and prices automatically adjust in equilibrium. In doing so, he tied together the theories of production, exchange, money, and capital." (*Encyclopaedia Britannica*)

The fundamental question that intrigued Walras: *how does a market in which many traders interact with each other, each one with some goods to sell and other goods to buy, reach a stable outcome (equilibrium) in which everyone is able to sell the goods they want to sell and buy the goods they want to buy*?

The mathematical model that he devised in order to find the answer to this question was inspired by the sort of markets that were common in the 19th century. In these markets, exchange ratios between goods could be determined by bargaining OR by auctioneers.

In his hypothetical model, auctioneers call out prices for items offered for sale. We study a simplified version of his model in Topic 2.

COURSE OVERVIEW

STRUCTURE AND LEARNING RESOURCES

01

INTRODUCTION

Foundations of classical monetary theory

02

THE REAL WALRASIAN ECONOMY

General equilibrium without money

03

THE WALRASIAN ECONOMY WITH MONEY

Integrating monetary exchange

04

SUMMARY

Key concepts and implications

Required Reading: Lewis and Mizen, Chapter 3 (skip section 3.6)

FOUNDATIONS OF CLASSICAL THEORY

Classical monetary theory rests on two pillars: the Walrasian system of general equilibrium and the Cambridge equation of exchange.

THE WALRASIAN SYSTEM

A framework for understanding how markets coordinate without central authority through price adjustments.

CAMBRIDGE EQUATION

Links the stock of money to the flow of transactions in the economy.

THREE CORE PRINCIPLES

1	2	3
MONEY IS A VEIL	**DICHOTOMY**	**NEUTRALITY OF MONEY**
Monetary exchange masks the fact that real economic activity is unaffected by the quantity of the medium of exchange.	Real variables (relative prices, real output, real interest rates) are independent of monetary variables (money supply, nominal prices).	Real variables are unaffected by changes in money supply, whilst nominal variables change proportionately.

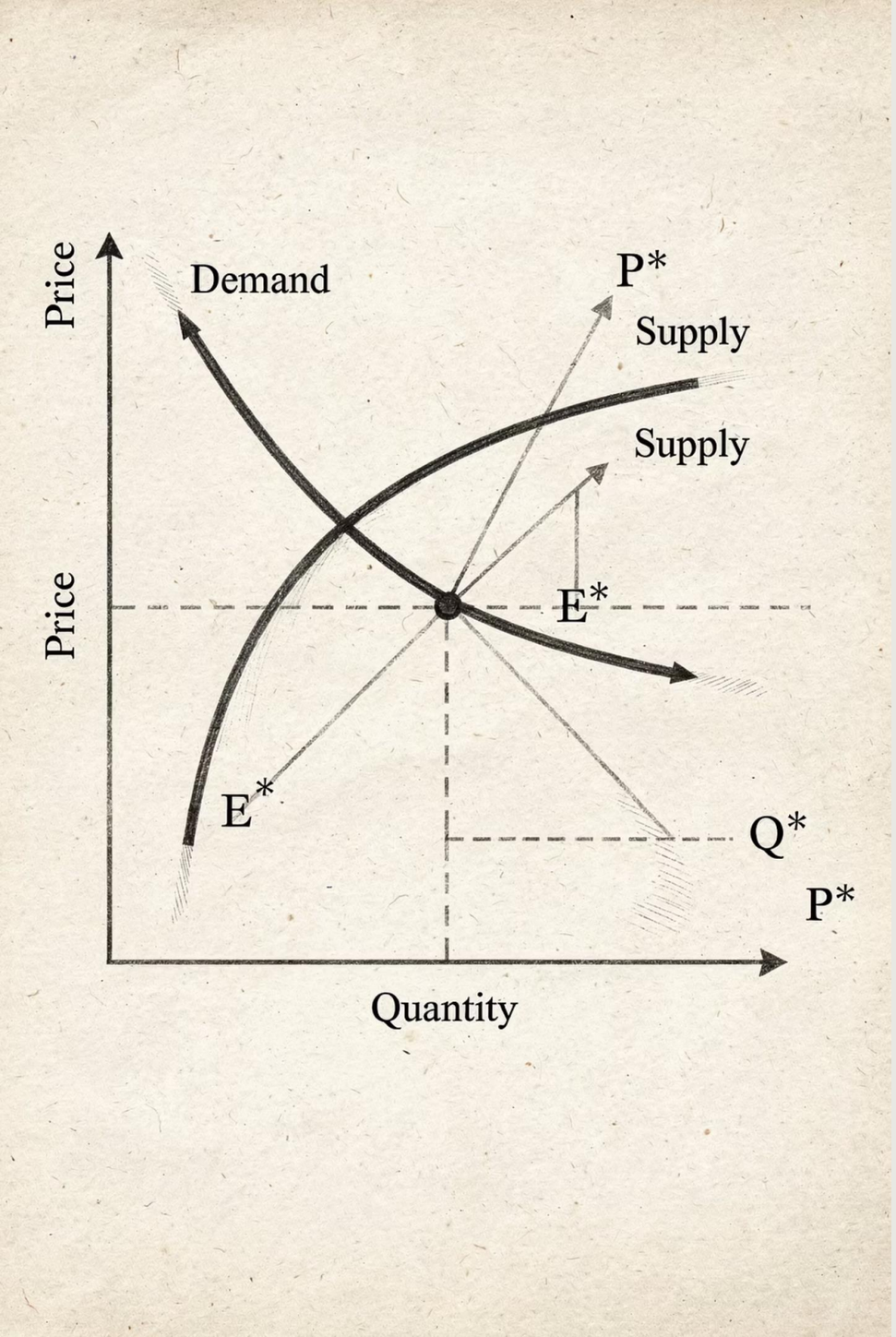
CHAPTER 1

THE REAL WALRASIAN ECONOMY



LÉON WALRAS (1834–1910)

French mathematician and economist, founder of general equilibrium theory. Walras posed a fundamental question: how does a market economy coordinate itself without central authority?



WALRAS' CENTRAL QUESTION QUESTION

> How does a market economy, with no central authority, coordinate itself so that an approximate match arises between the amounts demanded and supplied of each and every good?

Walras' Answer: Relative prices of goods adjust when supplies and demands do not match. As this occurs, supplies and demands move closer together until they equalise.

THE PURE EXCHANGE ECONOMY

The simplest version of Walras' model assumes a pure exchange economy.



PURE EXCHANGE

Total supply of each good is exogenous and constant.

Total demand varies according to relative prices.



PRODUCTION & EXCHANGE

Both supply and demand are endogenous variables adjusting to relative price changes.

ENDOWMENTS

In a pure exchange economy, each agent begins with an exogenous **endowment**: an arbitrary amount of each good initially possessed.

> **Key Property:** Endowments can be zero for some goods but never negative.

The motivation for exchange is that endowments are arbitrary. Agents trade away all or part of their initial endowment in exchange for other goods based on their preferences.

MATHEMATICAL NOTATION

Understanding the language of the Walrasian model:

SUPERSCRIPTS

Identify individual agents (e.g., agent *j*)

SUBSCRIPTS

Identify specific goods (e.g., good *i*)

This convention allows us to track both which agent and which good we are discussing in mathematical expressions.

AGENT ENDOWMENTS

Consider an economy with *X* agents and 3 goods. An abstract agent *j* has an endowment represented as a list:

$$(s_1^j, s_2^j, s_3^j)$$

where *s* represents an endowment.

The total supply of good *i* in the market is the sum of each agent's endowment:

$$S_i = s_i^1 + s_i^2 + \cdots + s_i^X$$

This sums over different agents (superscripts) whilst holding the good (subscript) constant.

DEMAND FUNCTIONS

Each agent's demand for a good depends on prices of all goods and their income:

$$d_i^j = d_i^j(P_1, P_2, P_3, y_N)$$

Agent *j*'s income is the market value of their endowment:

$$y_N^j = P_1 s_1^j + P_2 s_2^j + P_3 s_3^j$$

To add up different goods, a price system is used. This allows each good to be measured in value terms using a common yardstick—implicitly using a **unit of account**.

NOMINAL VERSUS REAL VALUES VALUES

NOMINAL INCOME

Measured in market value units. Can increase due to 'inflation' without any increase in real endowment.

Example: *y_N^j*

REAL INCOME

Measured in physical units of goods. Reflects actual purchasing power and real endowment.

Example: *y_N^j / P_1*



MARKET DEMAND FUNCTIONS

Individual demand functions can be aggregated into market demand functions:

$$D_i = d_i^1 + d_i^2 + \cdots + d_i^X = D_i(P_1, P_2, P_3, Y_N)$$

where nominal national income is:

$$Y_N = P_1S_1 + P_2S_2 + P_3S_3$$

YN represents nominal income because it uses the unit of account to measure and aggregate values of different goods. The same caveat about inflation applies.

HOMOGENEITY OF DEMAND

A demand function depends only on relative prices and real income, and is unaffected by proportionate changes in nominal prices:

$$d_i^j(P_1, P_2, P_3, y_N^j) = \quad d_i^j(kP_1, kP_2, kP_3, ky_N^j)$$

where *k* is any constant multiple. Therefore:

$$d_i^j(P_1, P_2, P_3, y_N^j) = \quad d_i^j\left(\frac{P_2}{P_1}, \frac{P_3}{P_1}, \frac{y_N^j}{P_1}\right)$$

Here, *y_N^j/P_1* is agent *j*'s real income, measured in equivalent units of good 1.

MARKET DEMAND HOMOGENEITY

Market demand functions inherit the homogeneity property:

$$D_i = \quad D_i \left(\frac{P_2}{P_1}, \frac{P_3}{P_1}, \frac{Y_N}{P_1}\right)$$

We define real national income as:

$$\frac{Y_N}{P_1} = S_1 + \frac{P_2}{P_1} S_2 + \frac{P_3}{P_1} S_3 \equiv Y$$

This is the sum of each good measured in units of good 1. The unit of account (numeraire) has changed from a fictitious measure of value to a concrete, physical object useful in the economy.



REAL INCOME EXAMPLE

Suppose an agent has endowment (2, 7, 3) and:

- One unit of good 2 is worth 0.5 units of good 1
- One unit of good 3 is worth 4 units of good 1

The agent's real income, measured in units of good 1, is:

$$2 + (7 \times 0.5) + (3 \times 4) = 2 + 3.5 + 12 = 17.5$$

GENERAL EQUILIBRIUM CONDITIONS

The Walrasian system is defined by equilibrium conditions for each good:

$$D_1\left(\frac{P_2}{P_1}, \frac{P_3}{P_1}, Y\right) = \quad S_1$$

$$D_2\left(\frac{P_2}{P_1}, \frac{P_3}{P_1}, Y\right) = \quad S_2$$

$$D_3\left(\frac{P_2}{P_1}, \frac{P_3}{P_1}, Y\right) = \quad S_3$$

Since *Y = S_1 + \frac{P_2}{P_1}S_2 + \frac{P_3}{P_1}S_3* and supplies are exogenous, the only endogenous variables are *\frac{P*

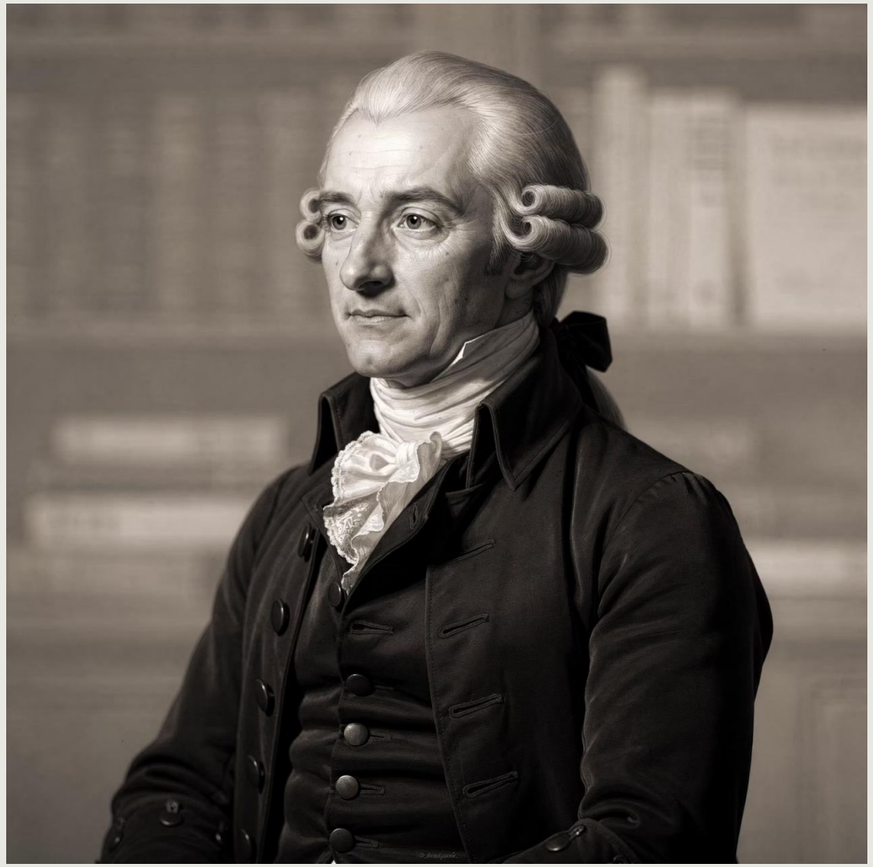
WALRAS' COORDINATION PROBLEM

The solution requires finding values of $\frac{P_2}{P_1}$ and $\frac{P_3}{P_1}$ such that each equilibrium equation is satisfied.

> Only 2 unknowns to solve 3 equations? This usually means multiple solutions can exist.

This apparent problem is resolved by Say's Law and Walras' Law.

JEAN-BAPTISTE SAY



SAY'S LAW

Jean-Baptiste Say (1767–1832) was a classical pro-free market economist known for coining Say's Law: **"Supply creates its own demand"**—an early defence of supply-side economics.

WALRAS' LAW

Walras expressed Say's Law algebraically:

$$P_1D_1 + P_2D_2 + P_3D_3 = P_1S_1 + P_2S_2 + P_3S_3$$

> The overall value of items supplied must equal the overall value of items demanded.

Implication: If two of the three markets are in equilibrium, then the third must be in equilibrium as well. Thus, one equation can be dropped, restoring equality of equations and unknowns.

SOLVING THE WALRASIAN SYSTEM

Due to Walras' Law, only two out of three equilibrium conditions need to be solved, restoring equality of equations and unknowns.

1

TWO EQUATIONS

Solve for two relative prices

2

TWO UNKNOWNS

$\frac{P_2}{P_1}$ and $\frac{P_3}{P_1}$

3

UNIQUE SOLUTION

If demand negatively related to relative price

HOMOGENEITY OF THE SYSTEM

If $\left(\frac{P_2}{P_1}, \frac{P_3}{P_1}\right)$ satisfies equilibrium, then multiplying each nominal price by a scale factor k also

$$\frac{kP_2}{kP_1}, \frac{kP_3}{kP_1}$$

> Because of homogeneity, the real Walrasian system cannot determine nominal prices.

This limitation is where the Classical Theory of Money becomes essential.

CHAPTER 2

THE WALRASIAN ECONOMY WITH MONEY

Suppose a fourth object exists, which we call 'money' and label as *M*.



UNIT OF ACCOUNT

Money serves as the unit of account. Goods prices *P_1, P_2, P_3* indicate how many units of *M* equal one unit of each good.



MEDIUM OF EXCHANGE

Money facilitates transactions between agents in the economy.

Money's own 'price' is defined as the amount of goods one unit of money can buy: *1/P_M*.

THE PRICE OF MONEY

One unit of money will buy $1/P_1$ units of good 1, $1/P_2$ units of good 2, and so on.

With multiple goods, we measure money's value against a fictitious **composite good**—a weighted index of actual goods (e.g., CPI, GDP deflator).

1/P_M

Amount of composite good one unit of money will buy

P_M

Units of money required to buy one unit of composite good

UNDERSTANDING UNITS OF MEASUREMENT

Units of *P_1, P_2, P_3*: money/good

Units of *1/P_1, 1/P_2*: goods/money

Units of *P_M*: money/composite good

Units of *1/P_M*: composite good/money

Consider the product *P_1 \times 1/P_M*:

$$\frac{M}{\text{good 1}} \times \frac{\text{composite good}}{M} = \frac{\text{composite good}}{\text{good 1}}$$

Thus *P_1/P_M* is the relative price of good 1 with respect to the composite good.

RELATIVE PRICES WITH MONEY

Relative goods prices are now defined as ratios of *P_M*:

$$\frac{P_1}{P_M}, \frac{P_2}{P_M}, \frac{P_3}{P_M}$$

Real national income, *Y*, is now:

$$Y = \frac{P_1}{P_M} S_1 + \frac{P_2}{P_M} S_2 + \frac{P_3}{P_M} S_3$$

The unit of account in the economy is now object *M*.

SUPPLY AND DEMAND FOR MONEY

SUPPLY OF MONEY

The supply *S_M* is exogenous, like that of goods.

Unlike other objects, *M* has no independent use and is not included in national income.

DEMAND FOR MONEY

Money is demanded for intermediate use as a medium of exchange, not for final use.

Demand depends on how much exchange takes place and how 'expensive' such exchange is.

THE QUANTITY THEORY OF MONEY

Demand for money is proportional to nominal income:

$$D_M = k \quad (P_1 S_1 + P_2 S_2 + P_3 S_3) = k \quad (P_M Y)$$

where k is a proportionality constant.

Demand for money increases if there are more goods or if the price of each good rises, resulting in an increase in the money price of the composite good. There are no other influences on demand for M.

CAMBRIDGE EQUATION OF EXCHANGE

Equilibrium in the market for money requires:

$$S_M = D_M \qquad S_M = kP_M Y$$

This is closely related to the Cambridge Equation of Exchange (CEE):

> Stock of money × velocity = nominal value of all transactions

Mathematically, QTM and CEE are identical if we set *k* as the inverse of the velocity of money.

QTM VERSUS CEE

CAMBRIDGE EQUATION (CEE)

True by definition. Reconciles the stock of money with the flow of transactions using that money.

QUANTITY THEORY (QTM)

Assumes velocity of money is constant and does not vary with economic variables like interest rates. This leads to the prediction that money demand is strictly proportional to nominal GDP.

KEYNESIAN ALTERNATIVE

Expresses demand as $D_M = k(i, \pi)P_M Y$ where i is nominal interest rate and π is inflation. The proportionality factor can change as other variables do.

EQUILIBRIUM WITH MONEY

Equilibrium must be achieved in all markets: 3 goods markets and the market for M.

Walras' Law in a monetary economy:

$$\sum_{i=1}^{3} P_i D_i + P_M D_M = \sum_{i=1}^{3} P_i S_i + P_M S_M$$

Important: Say's Law need not always hold since the value of all goods supplied need not equal the value of all goods demanded. This was Keynes' departure from classical theory.

THE COMPLETE SYSTEM

The Walrasian system with money:

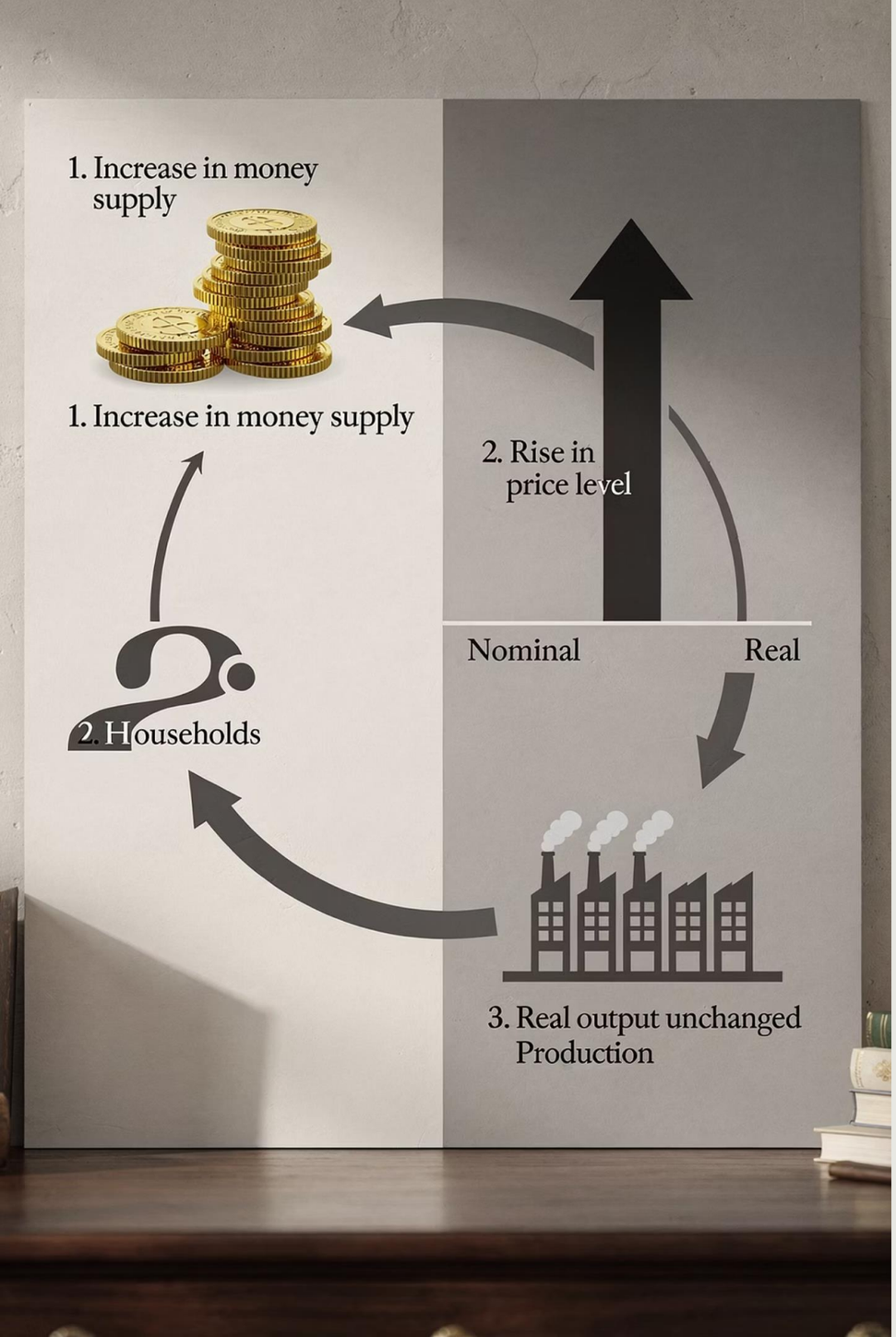
$$D_1\left(\frac{P_1}{P_M}, \frac{P_2}{P_M}, \frac{P_3}{P_M}, Y\right) = S_1$$

$$D_2\left(\frac{P_1}{P_M}, \frac{P_2}{P_M}, \frac{P_3}{P_M}, Y\right) = S_2$$

$$D_3\left(\frac{P_1}{P_M}, \frac{P_2}{P_M}, \frac{P_3}{P_M}, Y\right) = S_3$$

$$kP_M Y = S_M$$

Three goods equations independently solve for three relative prices. Then the price index *P_M* is solved from money market equilibrium.



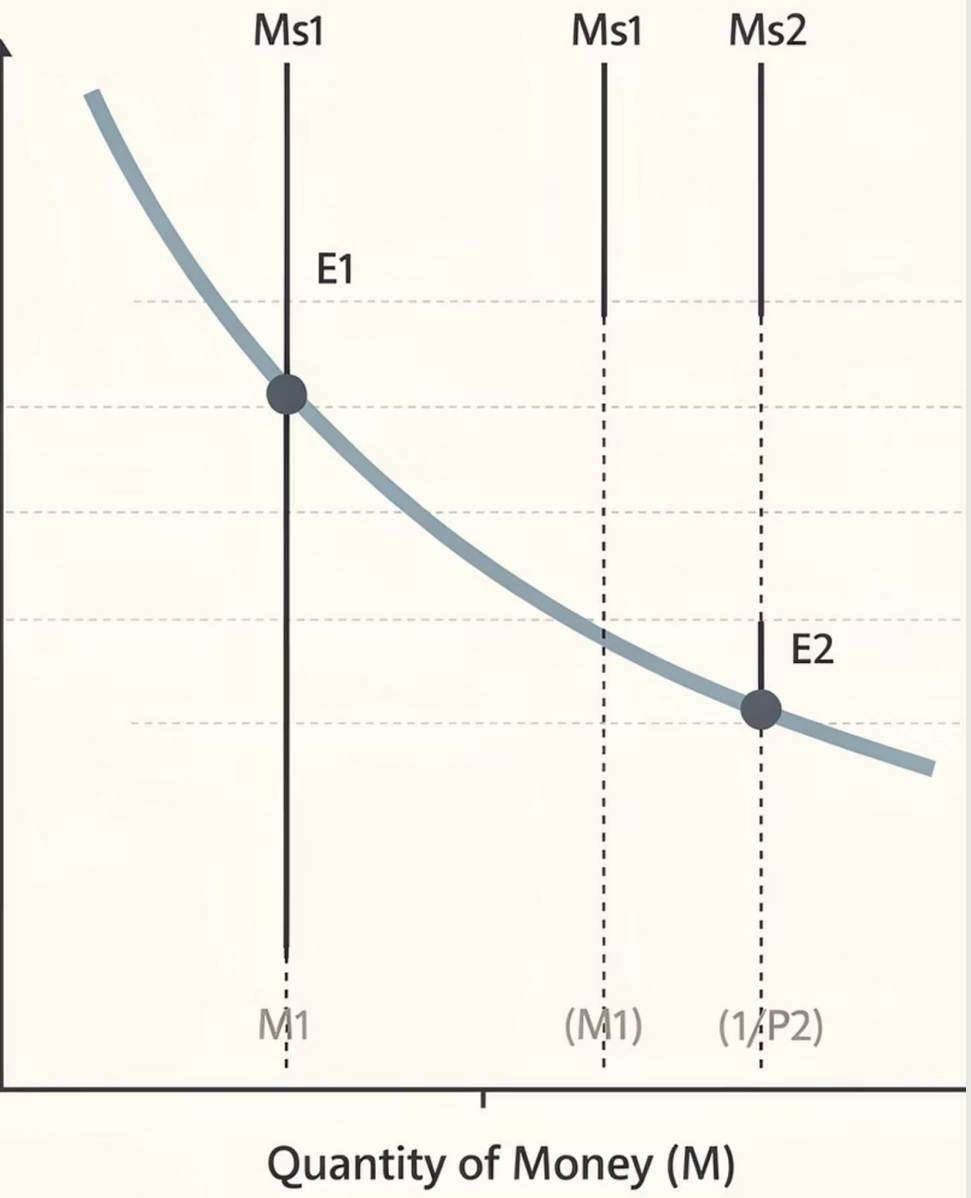
DICHOTOMY AND NEUTRALITY NEUTRALITY

DICHOTOMY OF REAL FROM NOMINAL

Relative prices $\frac{P_1}{P_M}$, $\frac{P_2}{P_M}$, $\frac{P_3}{P_M}$ and real GDP Y are solved from the 3 goods equations alone. The real economy is solved independently of the money market.

NEUTRALITY OF MONEY

If S_M doubles, then P_M will double, as will P_1, P_2, P_3. No effect on relative prices or on Y.



TRANSMISSION MECHANISM

Supply-demand analysis shows how changes in money supply lead to proportionate changes in prices.

Recall that $1/P_M$ is the price of money. From QTM, D_M increases in P_M, therefore it decreases in $1/P_M$.

An increase in money supply shifts S_M outwards, leading to a rise in P_M by an equivalent amount, so that S_M/P_M remains constant.

Mechanism: Higher money supply creates excess supply in the market for M. By Walras' Law, this implies overall excess demand for goods, leading to a rise in goods' prices and thus in P_M.

SUMMARY

KEY TAKEAWAYS

1 **FOUNDATIONS**

Classical theory of money is built on Walrasian foundations with the Quantity Theory of Money.

2 **HOMOGENEITY**

Exchange of goods depends only on relative prices, not nominal prices, reflected in the homogeneity property of demand functions.

3 **WALRAS' LAW**

Allows us to focus on $n - 1$ market-clearing equations, restoring equality of equations and unknowns.

4 **NOMINAL PRICES**

The Walrasian system of real exchange cannot determine nominal prices. Classical theory adds the QTM for this purpose.

5 **CLASSICAL CONCEPTS**

The combination of Walrasian system and QTM leads to dichotomy and neutrality of money.